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B.Sc Sem 4 — MIC

Que(1) :— State and Prove Dirichlet's Test theorem.

If for a Series Σu_n, the Sequence $\{S_n\}$ such that $S_n = u_1 + u_2 + \cdots + u_n$ is bounded and if $\{v_n\}$ is a positive monotonically decreasing sequence tending to zero, then the Series $\Sigma u_n v_n$ is convergent.

Proof :— We write $\alpha_n = u_1 v_1 + u_2 v_2 + u_3 v_3 + \cdots + u_n v_n$

then
$$\alpha_n = u_1 v_1 + u_2 v_2 + u_3 v_3 + \cdots + u_n v_n$$
$$= S_1 v_1 + (S_2 - S_1) v_2 + (S_3 - S_2) v_3 + \cdots + (S_n - S_{n-1}) v_n$$
$$= S_1 (v_1 - v_2) + S_2 (v_2 - v_3) + \cdots + S_{n-1} (v_{n-1} - v_n) + S_n v_n$$
$$= \sum_{k=1}^{n-1} S_k (v_k - v_{k+1}) + S_n v_n \quad \text{—— (1)}$$

Now, Since the Sequence $\{S_n\}$ is bounded and also the sequence $\{v_n\}$ is positive and monotonically decreasing therefore by the preceding theorem.

$\sum_{k=1}^{n-1} S_k (v_k - v_{k+1})$ tends to a finite limit as $n \to \infty$

Also Since $v_n \to 0$ as $n \to \infty$ and since $\{S_n\}$ is bounded, therefore $S_n v_n \to 0$ and $n \to \infty$

Hence from (1) We find that α_n tends to a finite limit as $n \to \infty$

Que(2) :— State and Prove Abel's Test theorem.

If the Series Σu_n is Convergent and if $\{v_n\}$ is a positive monotonically decreasing sequence, then the Series $\Sigma u_n v_n$ Converges.

Proof :- We write $\alpha_n = u_1v_1 + u_2v_2 + u_3v_3 + \cdots + u_nv_n$

then $\alpha_n = u_1v_1 + u_2v_2 + u_3v_3 + \cdots + u_nv_n$

$$= S_1v_1 + (S_2 - S_1)v_2 + (S_3 - S_2)v_3 + \cdots + (S_n - S_{n-1})v_n$$

$$= S_1(v_1 - v_2) + S_2(v_2 - v_3) + \cdots + S_{n-1}(v_{n-1} - v_n) + S_nv_n$$

$$= \sum_{k=1}^{n-1} S_k(v_k - v_{k+1}) + S_nv_n \quad \text{——— (1)}$$

Since Σu_n is convergent. therefore the sequence $\{S_n\}$ is bounded. Also it is given that $\{v_n\}$ is positive and monotonically decreasing Hence.

$\sum_{k=1}^{n-1} S_k(v_k - v_{k+1})$ tends to a finite limit as $n \to \infty$

Again the sequence $\{v_n\}$ is monotinically decreasing and it is bounded below by zero, & Hence $\{v_n\}$ tends to limit. Also the sequence $\{S_n\}$ is convergent

Therefore S_nv_n tends to a finite limit as $n \to \infty$

Therefore it follows from (1) that $\alpha_n \to$ a finite limit as $n \to \infty$

Hence the series Σu_nv_n is convergent.

Que (3) :- Theorem :- Define Beta function.

Proof :- Beta function is defined by

$$B(l, m) = \int_0^1 x^{l-1}(1-x)^{m-1}\, dx$$

and Gamma function is defined by

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\, dx$$

where l, m, n are positive numbers. integral or fractional